A Data Partitioning-based Artificial Neural Network Model to Estimate Real-driving Energy Consumption of Electric Buses

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ABSTRACT
Reliable and accurate estimation of electric bus energy consumption is critical for electric bus operation and planning. But energy prediction for electric buses is challenging because of diversified driving cycles of transit services. We propose to establish a data-partition based artificial neural network model to predict energy consumption of electric buses at microscopic level and link level. The purpose of data partitioning is to separate data into charging and discharging modes and then develop most efficient prediction for each mode. We utilize a long-term transit operation and energy consumption monitoring dataset from Chattanooga, SC to train and test our neural network models. The microscopic model estimates energy consumption of electric bus at 1Hz frequency based on instantaneous driving and road environment data. The prediction errors of micro model ranges between 8% and 15% on various instantaneous power demand, vehicle specific power, bins. The link-level model is to predict average energy consumption rate based on aggregated traffic pattern parameters derived from instantaneous driving data at second level. The prediction errors of link-level model are around 15% on various average speed, temperature and road grade conditions. The validation results demonstrate our models’ capability to capture impacts of driving, meteorology and road grade on electric bus energy consumption at different temporal and spatial resolution.

Keywords: Electric bus, Artificial neural network, Energy consumption prediction
INTRODUCTION

Globally, the transportation sector consists of 1.3 billion on-road vehicles and consumes 79 quadrillion BTU of energy, which emit 5.7 gigatonnes of CO2, and emit other pollutants that contribute to 200,000 annual premature deaths (1). Transportation authorities are employing a portfolio of strategies to mitigate GHG emissions and their dependence on fossil fuels (2–4). Public transportation is considered one important strategy because of its benefits in conveying larger passenger volume in much less space than individual automobiles (5). Specifically, electrification of public transportation is a promising approach promoted by industry, public agencies and research communities in solving the traffic problem, saving energy and reducing air pollution (6). Transit agencies have been increasing the use of electric buses in their operating fleet, but despite the environmental benefits and rapid growth of electric buses in the global market, the electricity mileage range and long charging time limit their wide adoption and make it challenging to plan electric bus operation and route assignment (7). Reliable and accurate estimation of electric bus energy consumption can facilitate electric bus operation and planning (8). However, energy consumption prediction of electric buses is challenging due to their complicated driving cycles and a wide range of factors that could influence energy consumption. Additionally, the format of models can vary depending on temporal granularity of prediction.

State of the art review

There is a school of literature on energy estimation models of electric vehicles, though limited studies focused on electric buses. In energy consumption models, the temporal granularity refers to time duration of energy prediction output. Existing energy prediction models can be classified as microscopic and macroscopic models based on time duration used for prediction. Micro models predict energy consumption of an electric vehicle at every second, thus normally necessitating sensing of vehicle driving parameters such as speed and acceleration at each second. Macroscopic models can explore the relationship between energy consumption and characteristics of driving at an aggregated spatial and/or temporal span. The prediction time duration can range from 5 minutes, to daily average. The methodologies of those electric vehicle energy consumption models can be classified into physics-based and data-driven. Physics-based models follow some fundamental physics laws and perform system identification to model the dynamics and interactions of various vehicle/powertrain components to estimate energy consumption. Data-driven models rely on the exploration of statistical relationships between inputs and energy outputs; thus, they can avoid understanding the physical process of vehicle dynamics and powertrain operation.

Micro models can assess energy impacts of changes in vehicle dynamics at high frequency levels, thus, they are widely adopted in applications related to real-time optimal control of traffic involving electric vehicles. For example, studies on electric passenger vehicles include eco-driving to optimize electric vehicle trajectories for energy savings (9, 10); eco-routing to dynamically determine energy efficient routes (11, 12); integrated with traffic simulation for calculating energy consumption (13). These models are developed by either physical-based or data-driven methods. The independent variables are vehicle dynamics and components variables. Vehicle dynamics are factors that reflect the motion (including speed, acceleration, and tractive/brake torque) of a vehicle. The laws of physics govern the direct relationship between these factors and (kinetic) energy demanded by vehicles. Vehicle component parameters govern the operating states of key parts for propulsion (e.g., electric motors, mechanical transmissions), and energy flows within the energy storage and auxiliary subsystems, e.g. motor and transmission efficiencies, aerodynamic coefficients, etc. The most relevant study of micro model for electric bus is by Beckers et al. (14), who developed a physical-based energy consumption prediction model using data from one single real-world trip of one electric city bus. The physical-based model is derived based on principle of force and complemented by dedicated measurements to quantify parameters such as drag coefficients, tire rolling resistance, powertrain efficiencies, etc., for the specific electric bus. Although the model achieved high accuracy in discharging estimation, it failed in predicting regenerated energy because road grade was not considered as an input variable to the model. Additionally, since the validation data contains only one service trip of one bus, its application is limited. Similar physical-based powertrain models are created by other researchers, e.g. Halmeaho et al. (15), Brook et a. (16), etc.
Macro models can utilize aggregated traffic and vehicle information to estimate energy consumption of electric vehicle over a time span, thus they are adopted in applications in electric vehicle fleet management (17, 18), regional planning of charging infrastructure or electric vehicle adoption (19), etc. These macro models are based on data-driven methods, because it is impossible to apply fundamental physical laws and mimic energy flow over a period of time. There are several studies using macro models for electric bus energy prediction. Pamula et al (20) proposed a machine-learning macro model to predict energy consumption rate between stops of electric buses using a reduced number of bus trip parameters, such as trip duration, month of trip (as a proxy to weather), and elevation changes between stops. The validation with real-world data showed their model has a mean absolute percentage error compared with actual consumption rate at 7% for testing trips. Although the model approach is solid, the estimation does not consider traffic (i.e. transient driving conditions) between stops. In addition, the month index used to approximate the weather condition does not account for the weather variation within the month which is not captured in the model. Vepsalainen et al. (21) proposed a computationally efficient surrogate model to predict route-level average energy consumption rate for electric buses. The surrogate model is based on an electric bus energy simulation model and used to conduct sensitivity analysis to evaluate changes in energy consumption rate due to variations in vehicle attributes (e.g. battery resistance, auxiliary power, mass) and environment parameters (e.g. ambient temperature, headwind). But the surrogate model is not based on real-world driving data and road grade is not considered in the model. Additionally, the route-level energy prediction is specific to the given origin and destination pair and less useful in evaluating alternative routes.

Contribution of the work

Energy prediction for electric buses is challenging because of diversified driving cycles of transit services. Above literature review shows limited studies on energy consumption of electric buses. The limited micro models are mainly built on physics-based approaches, which require efforts in validations of vehicle attribute parameters. The limited macro models have constraints either on omitting important influencing variables or developing models without validation with real-world data. In this study, we propose to establish a data-partition based artificial neural network energy prediction model for electric bus at microscopic level and link level. The data partitioning is applied to separate data into charging and discharging mode and then develop prediction models for each mode. This approach has proven to improve prediction performance because the energy rate at charging and discharging are influenced by different factors and have different features. We utilize a long-term real-world measurement dataset with diversified topological and meteorological features. The models can be used to assess real-time energy consumption of electric vehicles and evaluate energy benefits of vehicle control strategies at 1Hz frequency and at every road link. The microscopic model first partitions 1Hz driving data based on instantaneous speed and acceleration and then develops neural network models to each of data component to maximize prediction performance. The link-level model is to predict average energy consumption rate based on traffic pattern parameters deriving from second-by-second vehicle trajectories. The link-level model can leverage detailed trajectory data to achieve high accuracy and to guide energy-oriented transit operations and route planning.

DATA PREPATION

Vehicle specifications and Data collection

The on-board driving and energy consumption measurement has been recorded since July 2018 and is an on-going effort. The data are collected from three battery-powered electric buses operated by Chattanooga Area Regional Transportation Authority (CARTA) in Chattanooga, Tennessee. On-board measurement equipment is deployed on the studied electric buses to collect driving, monitoring and maintenance data of vehicles and the data are uploaded to a cloud in a real-time manner that can be used to analyze the status of the fleet. A summary of the electric bus configuration is presented in Table 1. The buses are BYD K9S, which are manufactured by one of the largest electric vehicles manufacturers in the world, BYD Auto Co., Ltd.

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Table 1 Summary of sampled transit buses and specifications</strong></td>
<td></td>
</tr>
<tr>
<td>Seat Capacity</td>
<td>31 seats</td>
</tr>
<tr>
<td>---------------</td>
<td>----------</td>
</tr>
<tr>
<td>Model year</td>
<td>2016</td>
</tr>
<tr>
<td>Measurements (L/W/H)</td>
<td>12/2.5/3.2 meters</td>
</tr>
<tr>
<td>Acceleration (0-50 kph)</td>
<td>20 seconds</td>
</tr>
<tr>
<td>Top speed</td>
<td>96 kph</td>
</tr>
<tr>
<td>Range fully charged</td>
<td>250 km</td>
</tr>
<tr>
<td>Gross weight</td>
<td>18,000 kg</td>
</tr>
<tr>
<td>Max power output</td>
<td>180 kW</td>
</tr>
<tr>
<td>Peak torque to wheels</td>
<td>1,100 Nm</td>
</tr>
<tr>
<td>Drive batteries</td>
<td>Li-Ion batteries</td>
</tr>
<tr>
<td>Battery Size</td>
<td>350 kWh</td>
</tr>
</tbody>
</table>

The collected data include data about real-time GPS locations (i.e. latitude, longitude), vehicle activities (instantaneous speed, acceleration and RPM), energy related parameters (i.e. energy consumption rate, state of charge) and meteorology data (ambient temperature and humidity). The data is retrieved at 1Hz frequency and can be synchronized with existing network performance index and elevation. The transit buses are running at pre-defined bus routes in Chattanooga metropolitan region, as shown in Figure 1 (left). The routes represent typical mountainous terrain patterns in the region, which is surrounded by Tennessee River and ridge-and-valley Appalachians. The driving cycles of transits running on those routes are shown in Figure 1 (right). The driving cycles have a speed up to 60~80 kilometer per hour (kph) and acceleration range -2 and 2 kph per second. This is comparable to the widely used transit cycle in research, orange county transit cycle, which has a maximum speed of 60 kph and acceleration range -1.3 to 1.4 kph per second.

Data infusion

Figure 2 illustrates the process of the data preparation. We first perform outlier detection on the raw data which will eliminate records with extreme and unlike values. The GPS-based longitude and latitude information is map-matched with a digital three-dimensional map to obtain elevation and road grade information for each record. Location and temporal specific humidity and temperature information is also obtained. All of the above information is merged into a single dataset that contains electric bus driving, operating as well as environment data.
We represent distributions of humidity, temperature and road grade in Figure 3. The distributions show that the dataset covers a wide range of humidity, temperature and road grade. The road grade ranges between -10% and 10%, which represents the mountainous terrain characteristics of the Chattanooga region.

Vehicle activity and energy consumption

The recorded 1 Hz data for vehicle speed (v), acceleration (a), road grade (grade) were combined to quantify instantaneous vehicle specific power (VSP) defined by Jimenez-Palacios (22). VSP is a proxy to instantaneous vehicle power demand and used in relevant studies to investigate impacts of power on vehicle energy consumption and emission (23, 24). The formulation is \( VSP_v = v_t (1.1a + 9.81g + 0.132) + 3.02 \times 10^{-4}(v_t)^3 \), with \( v_t \) (meter/second), \( a \) (meter/second²) and \( g \) (%) as instantaneous speed, acceleration and road grade. The unit for VSP is kW per ton. Figure 4 shows average fuel consumption (unit: kWh/second) and 95 percentage confidence intervals as a function of VSP. The results show a positive relationship between VSP and fuel consumption rate. The figure is our first attempt to understand energy consumption behavior of electric bus under different driving conditions. Although we observe a positive relationship between VSP and energy consumption rate, there are variations as well. Therefore, we should not rely on one single variable to estimate energy consumption rate of electric buses, but rather to leverage available data sources in this experiment to develop a comprehensive energy consumption model.
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Figure 1 Mean and 95% confidence interval of Instantaneous power rate (kW) as a function of vehicle specific power

**METHODOLOGY**

Power flows within the electric bus

The total energy consumption from a power source of a vehicle can be divided into energy used for overcoming changes in kinetic energy in vehicle’s motion \( \left( E_k \right) \), changes potential energy due to gravity \( \left( E_p \right) \), rolling resistance at tire \( \left( E_r \right) \), aerodynamic resistance of vehicle body \( \left( E_a \right) \), loss (internal energy loss) \( \left( E_l \right) \), and auxiliary consumption (e.g. A/C and all other accessories) \( E_a \). Thus, the required power during a time interval \( \Delta t \) (e.g. 1 second) \( P_t \) is calculated as

\[
P_t = \frac{E_k + E_p + E_r + E_a + E_l + E_a}{E_k + E_p + E_r + E_a + E_l + E_a}
\]

\[
= \frac{m \cdot a \cdot v + m \cdot v \cdot g \cdot \sin \varphi + m \cdot v \cdot \psi + m \cdot v^3 \cdot \zeta + P_{t+a}}{m \cdot v \cdot (a + g \cdot \sin \varphi + \psi + v^2 \cdot \zeta) + P_{t+a}}
\]

where \( a \) is vehicle acceleration during \( \Delta t \), \( v \) is average speed during \( \Delta t \), \( g \) is gravity constant, \( \varphi \) is road grade, \( \psi \) is rolling resistance coefficient constant, and \( \zeta \) is aerodynamic drag coefficient constant. Equations above show that instantaneous power \( P_t \) can be expressed as a function \( P_t = f(a, v, \varphi) \). In the above function, the instantaneous power can be negative when instantaneous acceleration \( a \) is negative, which is referred as regenerative braking. Under this driving condition, the battery is in charging energy status. This creates a challenge in energy estimation because the efficiency of charging, specified as regenerative factor \( \eta \), is not a fixed value and often influenced by the operating status of the vehicle. This challenge leads to partition data based on power charging and discharging, as well as driving status.

**Data partition**

Electric buses have two powertrain systems, the motor system is responsible to convert energy stored in the battery into kinetic energy for propelling the vehicle, and an active inverter system responsible for recovering the kinetic energy when vehicle brakes. We first divide data into accelerating (acceleration > 0 kilometer per hour per second, kph/s) and non-accelerating (otherwise) driving conditions. The energy recovering in non-accelerating status is challenging to predict because it depends on the recoverable kinetic energy and regenerative factor \( \eta \), i.e. percentage of available energy that are recovered. Through experiments under real-world driving conditions, Yang et al. (25) demonstrated that regenerative factor \( \eta \) is piecewise linearly related with vehicle speed with much higher \( \eta \) associated with vehicle speed below 18 kph (i.e. 5 m/s) and lower \( \eta \) with speed greater than 18 kph.
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Figure 5 Distribution of ECR (left); Different colors indicating accelerating or non-accelerating (right). The speed threshold of 5 m/s is equivalent to 18 kph.

We plot distributions of energy consumption based on driving status in Figure 5 (left). It shows that buses are accelerating, their batteries are in discharging mode for majority of time, which is as expected because the vehicle need power from the battery for propulsion. In non-accelerating driving, their batteries can be either charging or discharging modes but mainly in regenerative brake charging. In Figure 5 (right), we present distributions of energy consumption for non-accelerating data by discriminating with 18 kph (5 m/s) instantaneous speed threshold. We observe a pattern in the distributions, with charging rate falling in -100 to 0 kW driving with speed less than 18 kph, and rate of -200 to -100kW mainly for driving with speed higher than 18 kph. This piecewise pattern matched the relation between regenerative braking force and vehicle speed in literature.

Artificial Neural Networks Development for Microscopic Instantaneous Energy Consumption Prediction

With the analytical model presented in section 3.1, where the vehicle mass, rolling resistance coefficient, and drag coefficient are relatively constant and can be regarded as constants, we examine the functions approximated using artificial neural networks (ANN) training process to estimate instantaneous ECRs. ANNs are information processing systems that mimic how the human brain processes information (26). In ANNs, data processing components are called neurons, and neurons form a hierarchical structure consisting of input, hidden, and output layers. One neuron in a layer is a complex nonlinear combination of all neurons from the previous layer through coefficients and activation functions. Therefore, ANNs models can explore a wide range of nonlinear relationship between input variables and vehicle emissions rates. We select the variables listed in Table 2 as the features used in ANN model.

<table>
<thead>
<tr>
<th>Symbols</th>
<th>Descriptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t )</td>
<td>The index of second</td>
</tr>
<tr>
<td>( v_t )</td>
<td>The instantaneous speed at second ( t ) (kilometer / hour)</td>
</tr>
<tr>
<td>( a_t )</td>
<td>The instantaneous acceleration at second ( t ) (kilometer / hour / second)</td>
</tr>
<tr>
<td>( d_t )</td>
<td>The instantaneous engine speed at second ( t ) (round / min)</td>
</tr>
<tr>
<td>( w_t )</td>
<td>The ambient temperature at second ( t ) (Celsius degree)</td>
</tr>
<tr>
<td>( q_t )</td>
<td>The ambient humidity at second ( t ) (%)</td>
</tr>
<tr>
<td>( g_t )</td>
<td>The road grade at second ( t ) (%)</td>
</tr>
<tr>
<td>( VSP_t )</td>
<td>The instantaneous vehicle specific power at second ( t ) (kW/ton)</td>
</tr>
</tbody>
</table>

We partition the 2019 data (March and June 2019 data) into three subsets, i.e. accelerating driving, non-accelerating driving with speed threshold of 18 kph. Then, we randomly divided each subset into training and testing dataset to determine the optimal ANN model setup (neurons, hidden layer, activation function). We adopt the \( k \)-fold cross-validation method to prevent model over-fitting. This is a
sampling procedure where the dataset is randomly divided into \( K \) subsets of equal size. With the same neural network model setup, we conduct \( K \) training iterations. At each training iteration \( k \), the \( k \)th subset will be used as the testing dataset, and the remaining \( K-1 \) fold subsets will be combined to serve as the training set. Prediction performance for the \( k \)th training iteration will be recorded. After completing \( K \) training iterations, the prediction performance for the specific model setup (number of neurons, hidden layer, activation function) will be obtained by averaging performance from the \( k \) training iterations.

Neural network models with different setup will be compared based on their average prediction performance to identify the best performed model setup. We chose 5 as the training iteration \( K \) as suggested by literature \( (27) \). We chose mean absolute percentage error (MAPE) and coefficient of determination (\( R^2 \)) as the prediction performance metrics to compare neural network setups. Before applying ANN models, it is a common practice to normalize input and output data to speed up learning and lead to faster convergence in neural network solution algorithms. For an input variable \( x_i \), we normalize the \( i \)th record \( x_i \) as \( \frac{x_i - \min(x)}{\max(x) - \min(x)} \), where \( \min(x) \) and \( \max(x) \) correspond to the minimum and maximum values of that variable of all records. The normalized data is then be randomly divided into training and test datasets for neural network model development.

We adopt Mean Absolute Scaled Error (MASE) as a measure of forecast accuracy to compare the performance of the energy consumption rate estimation model. Although the Mean Absolute Percentage Error (MAPE) is easier to interpret, it is not suitable for sample data where values of dependent variable are largely zero or close to zero. Hyndaman and Koehlers \( (28) \) proposed the MASE to serve as a generally applicable forecast error measurement that is independent of the scale of data and has better predictable behavior when dependent variable is close to zero. MASE is calculated as following: \( MASE = \frac{1}{n} \sum_{j=1}^{n} \left( \frac{|P_i - \hat{P}_i|}{\min(|P_i - \hat{P}_i|)} \right) \), where \( P \) is observed energy consumption rate, \( \hat{P} \) is the estimated energy consumption rate, and \( n \) is the total number of observations in the testing dataset.

**Artificial Neural Networks Development for Link-based Energy Consumption**

The microscopic model requires instantaneous driving data, which are obtained either through large-scale vehicle monitoring program or simulation. But these methods are either costly or time-consuming to setup. Current technologies allow real-time acquisition of the average traffic speed of each link in a road network, and this average speed can be regarded as an equivalent approximate of the average speed of the vehicle when passing the link. In this study, we develop another neural network that uses aggregated traffic information to predict link-based energy consumption of electric buses.

To prepare training and testing data, we aggregated the time-sequenced vehicle driving data based on links and use them as the features in the ANN model (shown in Table 3). We adopt a similar procedure to normalize data and cross-validate data as discussed in microscopic model.

**TABLE 3 Description of Input Variables for Link-based Energy Consumption Model**

<table>
<thead>
<tr>
<th>Symbols</th>
<th>Descriptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v_l )</td>
<td>The average speed of the vehicle passing through link ( l ) (kilometer / hour)</td>
</tr>
<tr>
<td>( w_l )</td>
<td>The average ambient temperature of the vehicle through link ( l ) (Celsius degree)</td>
</tr>
<tr>
<td>( q_l )</td>
<td>The average humidity of a vehicle through link ( l ) (%)</td>
</tr>
<tr>
<td>( g_l )</td>
<td>The net road grade of the vehicle passing through link ( l ) (%)</td>
</tr>
<tr>
<td>( v_l^2 )</td>
<td>The squared link average vehicle speed on link ( l )</td>
</tr>
<tr>
<td>( v_l^3 )</td>
<td>The cubic link average vehicle speed on link ( l )</td>
</tr>
</tbody>
</table>

In neural network model development, one model setup is a combination of decisions on 1) which independent variables to include, 2) number of hidden layers and neurons in each layer, 3) format of activation function. The number of neurons in hidden layers are generally determined by using a trial and
error method (29), but one commonly used guideline in determining upper limit for neuron in hidden layers \( N_h \) is \( N_h \leq 2N_l + 1 \), with \( N_l \) to be number of independent variables. Activation function is responsible for transforming the set of neurons in one layer into a given neuron or output in the next layer. Most neural network models use nonlinear activation functions for the neuron in hidden layers to allow the model to learn complex structures in the data. Commonly seen nonlinear functions include sigmoid \( \frac{1}{1+e^{-x}} \), tangent \( \frac{2}{1+e^{-2x}} - 1 \) and rectifier \( \max(0,x) \) are tested in this study.

RESULTS AND DISCUSSION

We conduct model selection process and present the optimal model configurations and prediction performance metrics for the ANN models for link-based model and microscopic models based on data collected in year 2019 (Table 4). For microscopic models, the acceleration sub-model has a better \( R^2 \) and MASE results compared with the non-acceleration sub-models, followed by the link-based model.

<table>
<thead>
<tr>
<th>TABLE 4 Optimal Model Configuration and Performance Metric</th>
</tr>
</thead>
<tbody>
<tr>
<td>Layers</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Micro Model</td>
</tr>
<tr>
<td>Acc.</td>
</tr>
<tr>
<td>Non-Acc ( v&lt;18 \text{ kph} )</td>
</tr>
<tr>
<td>Non-Acc ( v&gt;18 \text{ kph} )</td>
</tr>
<tr>
<td>Link-based model</td>
</tr>
</tbody>
</table>

We test predictions of electricity consumption and evaluate its prediction accuracy at different driving conditions and trip durations. Figure 6 (left) presents absolute percentage errors of microscopic energy consumption as a function of vehicle specific power (VSP). The bar within each box corresponds to the mean absolute percentage error and the two sides of a box show the 1\textsuperscript{st} and 3\textsuperscript{rd} quantiles of the errors. The results show that errors of microscopic model reduce as VSP increases. The error does not change when VSP is greater than 15 kW/ton. High VSP (particularly larger than 15 kW/ton) corresponds to driving at high acceleration and climbing high hill, whose driving conditions are homogenous. Low VSP corresponds to more diversified driving conditions, such as high speed with low/negative acceleration, low speed with low/moderate acceleration etc. Therefore, our results verify that energy prediction contains more noise under diversified driving conditions.

Figure 6 (left) Boxplot of absolute percentage error (%) for microscopic energy consumption as a function of vehicle specific power. The bar corresponds to median and two sides of box are 1\textsuperscript{st} and 3\textsuperscript{rd} quartiles. The circle within each box represents mean.

Figure 6 (right) Mean and 95% confidence interval of absolute percentage errors of microscopic energy prediction at different trip duration.
We aggregate predicted electricity consumption rate of microscopic model over different trip durations (10 to 60 minutes) and assess prediction errors against actual energy consumption in Figure 6 (right). The mean absolute percentage error (MAPE) reduces from 10% in 10-minute trip to 8% in 60-minute trip. But the rate of reduction diminishes as duration increases and does not change significantly for trips with 40 minutes or more.

We apply the ANN micro model developed using 2019 data to a dataset collected in March 2020. The purpose is to validate the model and test overfitting possibility when it is applied outside of the original training and testing dataset. In Figure 7, we compare the absolute percentage error for predictions of the artificial neural network model and non-neural network linear regression model as a function of trip duration and vehicle specific power using 2019 (original training data) and 2020 (validation data) dataset. The results show that the mean absolute percentage errors and confidence intervals of artificial neural network model are consistently lower than those of linear regress model using either the original training data collected in 2019 or validation data collected in 2020. Though, the differences seem to diminish as trip duration increase in all cases.

Using the original training data collected in 2019, MAPE of ANN model is reduced from 10% to 8% while from 15% to 12% for linear regression model when trip duration increases from 5 minutes to 60 minutes. This is expected as shorter trip duration normally constitute more dynamic traffic and driving conditions thus neural network model can better capture changes in fuel consumption using complex model formats. When the trained model is applied data in year 2020, we observe that the MAPEs are higher than those using original training data for all trip durations. This shows possible overfitting characteristics of both neural network and linear regression model. The overfitting occurs when our models learn the detail and noise specific to the training data collected in 2019, which leads to higher prediction error when new data (2020 data) are applied to the models. However, it is promising to see that ANN models outperform linear regression when it comes to increase of error due to overfitting. This demonstrates ANN model’s capability in capturing non-linearities and complex relationship, which help reduce overfitting impacts at new data. When discriminating by VSP, neural network model outperforms linear regression model in all of the VSP bins for the origin training data collected in 2019. The improvements in prediction error are significant in low VSP areas (VSP < 3 kW/ton) for neural network model. When the developed model is applied to new data collected in 2020, we also observed increasing errors due to overfitting to the training data. But we observe the overfitting impacts are much less on ANN models than in linear regression, which is similar to the observation when prediction errors are accumulated at different trip duration levels.

![Figure 7 Mean absolute percentage error and 95 percentage confidence intervals for predictions of artificial neural network (ANN) model and linear regression model with the same independent variables as a function of trip duration (left) and vehicle specific power (right).](image)

The link-based model predicts average energy consumption rate (kWh/km) of electric bus at each road link based on traffic patterns and road environment on that specific link. Most of the links are between 1 and 5 miles with trip duration below 10 minutes. To better understand the model performance
under the impact of factors including traffic flow speed, ambient temperature and road grade, we present boxplot of link-based model errors with respect to these factors (Figure 8). We notice that for the traffic average speed range of 40-80 kph, the prediction error is slightly higher than other speed ranges. Trips with average speed ranging 40-80 kph can have a diversified driving with a mix of high speed, low speed driving and transient conditions. This increases complexity in energy prediction because the model has to be sensitive to changes of traffic patterns and capable of capture transient conditions in estimation. Figure 8 shows the ambient temperature of transit operation clusters around 10 and 25 Celsius, which correspond to morning/evening and middle day temperature. By discriminating trips based on temperature at these two categories, we do not observe significant differences in prediction error distributions. This demonstrates our energy prediction model’s capability to capture impacts of temperature on energy consumption which results in similar prediction error distribution of the two temperature categories. When link-based predictions are differentiated by road grade, links with flat or small hills generate the lowest prediction error followed by downhill and then uphill driving.

Figure 8 Boxplot of absolute percentage error (%) for link-based energy consumption discriminating by trip average speed, temperature and road grade. The bar corresponds to median and two sides of box are 1st and 3rd quartiles.

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REFERENCES


